Critical Conductance of a Mesoscopic System: Interplay of the Spectral and Eigenfunction Correlations at the Metal-Insulator Transition

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We study the system-size dependence of the averaged critical conductance $g(L)$ at the Anderson transition. We have: (i) related the correction $\delta g(L) = g(\infty) - g(L) \propto L^{-y}$ to the spectral correlations; (ii) expressed $\delta g(L)$ in terms of the quantum return probability; (iii) argued that $y = \eta$ – the critical exponent of eigenfunction correlations. Experimental implications are discussed.

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Energy-level statistics and wavefunction correlations are two basic issues in the theory of disordered electron systems at the Anderson metal-insulator transition. So far, however, progress here has been largely limited to scaling arguments in combination with numerical simulations – for lack of any parameter at the critical point. Our interest in this problem owes much to the recent works [1,2], where a remarkably simple relation between two fundamental characteristics of the critical system, the spectral compressibility and the critical exponent of the eigenfunction correlations, was *derived* analytically.

In the present paper we link the basic idea in [1,2] with the problem of long-term kinetic correlations at the metal-insulator transition. Specifically, we address the question as to the mechanism of the system-size (L) dependence of the ensemble-average critical conductance $g(L)$. In accordance with the general principle of scale invariance at the critical point, the critical conductance at the Anderson transition [3] converges at large L to a scale-invariant value $g(\infty) \sim 1$ (in units of e^2/h). What we are interested in is thus the correction $\delta g(L) = g(\infty) - g(L)$, which breaks the macroscopic scale invariance and behaves as

$$\delta g(L) \sim (L_c/L)^y \tag{1}$$

with a critical exponent $y > 0$. The microscopic length scale L_c is defined by the condition $\delta g(L_c) \sim 1$ and for the conventional Anderson transition in three dimensions (d) coincides with the mean free path l. Most of our discussion will be focused on the $2d$ case; namely, on the integer quantum Hall (QH) critical point. The dissipative conductance at the QH transition obeys Eq. (1) with the length L_c given by $\ln(L_c/l) \sim g^2(l)$; so that, in principle, the "elementary block" scale L_c may be much larger than l. For concreteness, however, we assume in what follows that $g(l) \sim 1$ and $L_c \sim l$. This is the case, e.g., at the plateau transition in the lowest Landau level. We seek a universal relation between the exponent y and other critical exponents, independent of the microscopic details.

Our main result is the following. We argue that

$$\delta g(L) \propto \left[\int_{\tau}^{L^2/D} dt P(0,t) \right]^{-1} , \tag{2}$$

where $P(0,t)$ is the quantum return probability in time t, τ the scattering time, $D \propto L^{2-d}$ [3] the long wavelength diffusion coefficient (independent of L in the $2d$ case). The above integral is familiar from the weak-localization theory [3]: in the simplest case of time-reversal symmetry, the finite-size correction to $g \gg 1$ in $2d$ is proportional to it and grows with L as $\ln(L/l)$. At the critical point the integral diverges more strongly – in a power-law manner. What we show is that at the transition the scale-dependent correction to the conductance is given by the *inverse* of the integral. It follows that the exponent y is not an independent one, but is equal to the critical exponent of eigenfunction correlations η [4], which controls behavior of $P(0,t) \propto t^{-1+\eta/d}$ at large t.

We start by defining two disorder-averaged two-particle correlators

$$P_\omega(r) = \frac{2\pi}{\rho} \langle \rho_\epsilon(0,\mathbf{r}) \rho_{\epsilon+\omega}(\mathbf{r},0) \rangle , \tag{3}$$
$$M_\omega(r) = \frac{2\pi}{\rho} \langle \rho_\epsilon(0,0) \rho_{\epsilon+\omega}(\mathbf{r},\mathbf{r}) \rangle ,$$

where the one-particle spectral function at energy ϵ reads $\rho_\epsilon(\mathbf{r},\mathbf{r}') = \sum_\alpha \delta(\epsilon - \epsilon_\alpha) \psi_\alpha(\mathbf{r}) \psi^*_\alpha(\mathbf{r}')$ and α labels eigenstates in a given realization of disorder. Throughout the paper we neglect the ϵ dependence of disorder-averaged quantities, in particular of the averaged density of states $\rho = \langle \rho_\epsilon \rangle$, where $\rho_\epsilon = L^{-d} \int d^d\mathbf{r} \rho_\epsilon(\mathbf{r},\mathbf{r})$, L^d is the system volume. The normalization of the first ("kinetic") correlator $\int d^d\mathbf{r} P_\omega(r) = 2\pi\delta(\omega)$ reflects particle number conservation, while $\int d^d\mathbf{r} M_\omega(r) = 2\pi\Delta^{-1}(1 + R_\omega)$ is expressed in terms of the correlator of spectral fluctuations

$$R_\omega = \frac{1}{\rho^2} \langle \delta\rho_\epsilon \delta\rho_{\epsilon+\omega} \rangle . \tag{4}$$

In the above, $\Delta = 1/\rho L^d \to 0$ is the mean level spacing.

We now write down the identity

$$\frac{1}{\Delta} \frac{\partial^2 R_\omega}{\partial \Lambda^2} = \frac{\partial^2}{\partial \omega^2} \left\langle \sum_\beta \left(\frac{\partial \omega_{\alpha\beta}}{\partial \Lambda} \right)^2 \delta(\omega_{\alpha\beta} + \omega) \right\rangle \tag{5}$$
$$+ \frac{\partial}{\partial \omega} \left\langle \sum_\beta \frac{\partial^2 \omega_{\alpha\beta}}{\partial \Lambda^2} \delta(\omega_{\alpha\beta} + \omega) \right\rangle ,$$

where Λ is an arbitrary parameter characterizing the Hamitonian of the system $H(\Lambda)$ and $\omega_{\alpha\beta} = \epsilon_\alpha - \epsilon_\beta$. We specify the parameter Λ by considering $\partial^2 R_\omega/\partial\Lambda^2$ a second-order response of the system to the small ($\Lambda \to 0$) perturbation $\delta H(\Lambda) = \Lambda e^{i\mathbf{kr}}$. Doing so we in fact introduce a $\mathbf{k}$-dependent spectral correlator $R_\omega(k,\Lambda)$. Next we define its Fourier transform $\mathcal{R}_\omega(r,\Lambda) = \int \frac{d^d\mathbf{k}}{(2\pi)^d} e^{-i\mathbf{kr}} R_\omega(k,\Lambda)$ and expand in powers of Λ. To second order in Λ [Eq. (5)] the expansion of $\mathcal{R}_\omega(r)$ readily yields

$$\frac{1}{\Delta}\frac{\partial^2 \mathcal{R}_\omega(r)}{\partial\Lambda^2} = -\frac{1}{\pi}\frac{\partial^2 M_\omega(r)}{\partial\omega^2} + \frac{2}{\rho^2\Delta}\frac{\partial}{\partial\omega}\int\frac{d\omega'}{\omega'}\left\langle \rho_\epsilon(0,\mathbf{r})\rho_{\epsilon+\omega'}(\mathbf{r},0)\left(\rho_{\epsilon+\omega+\omega'} - \rho_{\epsilon+\omega}\right)\right\rangle \ . \tag{6}$$

We decompose the three-particle correlator in the second line of this equation according to the following pattern

$$\rho\left\langle \delta\rho_{\epsilon_1}(0,\mathbf{r})\delta\rho_{\epsilon_2}(\mathbf{r},0)\delta\rho_{\epsilon_3}\right\rangle \simeq \left\langle \delta\rho_{\epsilon_1}(0,\mathbf{r})\delta\rho_{\epsilon_2}(\mathbf{r},0)\right\rangle\left(\left\langle \delta\rho_{\epsilon_1}\delta\rho_{\epsilon_3}\right\rangle + \left\langle \delta\rho_{\epsilon_2}\delta\rho_{\epsilon_3}\right\rangle\right) \ . \tag{7}$$

Note the extreme "non-Gaussian" character of the correlations: the irreducible three-particle average is represented as a product of two pairwise correlators. The crucial assumption behind this decoupling is that the correlations can be separated into fast and slow (in energy space) pieces: namely, the *spectral* correlations [Eq. (4)] decay rapidly on an energy scale which vanishes in the thermodynamic limit, whereas the *eigenfunction* correlations [Eq. (3)] decay slowly according to the diffusion law. In effect, if one puts $r = 0$, Eq. (7) constitutes precisely what was incorporated in the Langevin description (and was called a "t-space decoupling") in [1,2]. When substituted into Eq. (6), it gives, for the last term in the r.h.s.,

$$\frac{1}{\pi\Delta}\int\frac{d\omega'}{\omega'}P_{\omega'}(r)\frac{\partial}{\partial\omega}(R_{\omega+\omega'} - R_{\omega-\omega'}) \ . \tag{8}$$

The decomposition above is a peculiar property of the average (7) and we proceed to discuss the conditions under which it is legitimate (the conditions will equally apply to the method of [1,2]). If R_ω would be characterized by a single scale on which it falls off *sharply*, the separation of the fast and slow variables would simply require that $P_\omega(r)$ be a smooth function of ω on this scale. The spectral correlations (in closed systems) indeed start to decay on the smallest scale of Δ; specifically, $R_\omega \propto \omega^{-2}$ at $\Delta \lesssim |\omega| \lesssim E_c$ [5], where $E_c \sim g\Delta$ is the Thouless energy, and continue to decay at larger $|\omega|$. However, the argument that the energy-level correlations are short ranged at $\Delta \to 0$ may be illusive, since R_ω falls off at $|\omega| \gtrsim E_c$ in a power-law manner, which necessitates a power counting. What really matters to the justification of the above decoupling is *how* fast R_ω vanishes at large $|\omega|$. E.g., one sees that if R_ω decays slower than ω^{-1}, all frequencies in the range $|\omega'| \lesssim |\omega|$ will give roughly the same contribution to the integral over ω' in Eq. (8) – in contrast to what the decoupling implies. The sought conditions can thus be formulated as follows: (i) R_ω falls off with increasing $|\omega|$ not slower than ω^{-1}; (ii) $P_\omega(0)$ does not decrease with $|\omega|$ faster than ω^{-1}. Here and below we mean the frequency dependence in the diffusive regime $E_c \lesssim |\omega| \lesssim \tau^{-1}$.

Clearly, the decomposition in terms of the pairwise correlators fails in the insulating phase, where $P_\omega(r)$ is singular at $\omega = 0$ [see condition (ii)] even in the thermodynamic limit. So the real question is about accuracy of the decoupling in a system with finite g, where the condition on $P_\omega(r)$ is not severe but the behavior of R_ω [see condition (i)] becomes crucial. It is worthwhile to emphasize at this point that the validity of the decoupling is not related directly to the expansion in powers of $1/g$ and the above conditions can be satisfied (at $\Delta/|\omega| \ll 1$) even if $g \sim 1$. On the other hand, it is important that the diagrammatic analysis [1] up to three-loop order does conform to these conditions. Consider particular examples. The frequency dependence of R_ω becomes steeper as the localization effects get stronger and so does it with decreasing dimensionality. In the $3d$ case, at $g(l) \gg 1$, R_ω decays too slowly – as $|\omega|^{-1/2}$ [6] – which means that the decoupling certainly fails deep in the metallic phase. In the critical regime of the metal-insulator transition, R_ω behaves as $|\omega|^{-x}$ at $|\omega| \gtrsim \Delta$ and how x is related to other critical exponents is still an open problem [7]. However, the remarkable point to notice is that the decoupling which fails in the metallic phase may work at the transition, provided $x > 1$. In $2d$, R_ω falls off at $g \gg 1$ as $|\omega|^{-1}$ [8], which is a marginal case in respect of the validity of the decoupling (meaning logarithmic accuracy). The ω^{-1} behavior of R_ω at large g implies that $x > 1$ at the QH transition, which is one of the above conditions. Now, we must also be concerned about how $P_\omega(r)$ behaves as $\omega \to 0$. At the transition $P_\omega(r)$ diverges at small ω, down to $|\omega| \sim \Delta$, as $|\omega|^{-\eta/d}$ [4]. To decouple the spectral and eigenfunction correlations in $2d$, one needs $\eta < 2$ ($\eta = 2$ corresponds to an infinitely sparse fractal). At the QH critical point $\eta \simeq 0.4$ [4]). We conclude that the decoupling is valid at the QH transition [9].

From now on we specialize to the QH transition. The net result of the decoupling (7) is that the expansion of $\mathcal{R}_\omega(r)$ in Λ for any given ω and r yields just $M_\omega(r)$ and $P_\omega(r)$. It is convenient to transform to the time representation – introducing $M(r,t)$ and $P(r,t)$ – where the relation between the two functions assumes a particularly compact form:

$$tM(r,t) - 2K\int_0^t dt' P(r,t') = \frac{1}{2t}\frac{\partial^2\mathcal{K}(r,t)}{\partial\Lambda^2} \ . \tag{9}$$

Here we have also introduced the dimensionless Fourier transform $K(t) = \Delta^{-1}\int d\omega e^{-i\omega t}R_\omega$ and a similar transform $\mathcal{K}(r,t)$ for $\mathcal{R}_\omega(r)$. It is worth emphasizing that the

validity of the decoupling (the condition (i) above) requires $K(t)$ to be constant in the thermodynamic limit [10]. For our purposes, we thus may write $R_\omega = K\delta(\omega/\Delta) + (\Delta/\Sigma)F(\omega/\Sigma)$, where $\Sigma = \tau^{-1}$, $F(x)$ is a dimensionless function of order unity. It is the last term that is crucial to the analysis of the finite-size scaling. Clearly, it violates the condition (i); nevertheless, Eq. (9) is useful if we are interested only in the critical exponents – not the exact (nonuniversal) shape of $F(x)$. Indeed, at $r = 0$ we recover the relation [2]

$$K = \lim_{t/\tau\to\infty} \frac{1}{2} tP(0,t) \left(\int_0^t dt' P(0,t') \right)^{-1} , \qquad (10)$$

which, taken at finite t/τ, allows to estimate the small t-dependent term in $K(t)$. In this way we find that $K(\infty) - K(t) \sim (\tau/t)^{\eta/2}$ (i.e., $F(x)$ diverges as $x^{-1+\eta/2}$ at $x \to 0$). Note that in going to the last step we have neglected the parametric correlations in the r.h.s. of Eq. (7) (while in [1] they were omitted by construction), which needs explanation. We first rewrite $\partial^2\mathcal{K}(r,t)/\partial\Lambda^2$ at $r = 0$ as the response $\partial^2 K(t)/\partial u^2$ to the application of a white-noise random potential $W(\mathbf{r})$ with the correlator $\langle WW\rangle = u^2\delta(\mathbf{r})$. This additional random potential merely renormalizes the scattering rate. It follows that

$$\frac{\partial^2\mathcal{K}(0,t)}{\partial\Lambda^2} = 4\pi\rho \frac{\partial K(t)}{\partial\Sigma} . \qquad (11)$$

(recall that, for simplicity, we keep ρ independent of Σ, even at $g \sim 1$). Hence the starting idea in [1], about the statistical equivalence of all members of the ensemble of Hamiltonians, amounts to ignoring the dependence of $K(t)$ on Σ. One can easily estimate the omitted contribution to $K(t)$: it is $\sim (\tau/t)^{1+\eta}$, which is indeed smaller than the leading correction obtained above. Now we can argue in the same way to make sure that if t is large enough, the parametric correlations can be neglected at finite r as well. The large-t limit means $t \gg \max\{\tau, r^2/D\}$, so that as r increases it needs more time for the parametric correlations to be forgotten. It is also instructive to see how in $2d$ the first terms in the expansion of M, P, and K in powers of g^{-1} cancel in Eq. (9). Note that $M(r,t)$ and $P(r,t)$ can be split into short and long ranged (in real space) parts. Specifically, $M(r,t) \simeq 2\pi\rho\beta(0,t) + f^2(r)p(0,t) + (2\pi\rho)^{-1}\int_0^t dt' p(r,t')p(r,t-t')$, $P(r,t) \simeq 2\pi\rho\beta(r,t) + p(r,t)$, and $K \simeq (4\pi g)^{-1}$. Here $\beta(r,t)$ describes ballistic motion at $t \lesssim \tau$ and vanishes at larger t, $\int dt\beta(r,t) = f^2(r)$, $f(r) = e^{-r/2l}J_0(k_F r)$ is the Friedel function (k_F is the Fermi momentum), and $p(r,t)$ is the retarded Green function of the classical diffusion equation. For concreteness, we mean a system with completely broken time-reversal symmetry. Since $K \sim g^{-1}$, we need next order terms in $M(r,t)$ as compared to $P(r,t)$ – hence the two-diffuson term above. One can see that in the limit $t \gg \max\{\tau, r^2/D\}$ all the terms cancel out in the l.h.s. of Eq. (9). Despite the simplicity, it is a useful check.

Now we write the correlators in the scaling form which they assume at the QH transition: $M(r,t) = \frac{1}{Dt}\mathrm{M}(\frac{r^2}{Dt})$, $P(r,t) = \frac{1}{Dt}\mathrm{P}(\frac{r^2}{Dt})$, where both $\mathrm{M}(x)$ and $\mathrm{P}(x)$ behave as $x^{-\eta/2}$ at $x \ll 1$ with $\eta \simeq 0.4$ [4]. The hydrodynamic form of the scaling functions implies that $r \gg \lambda$ and $t \gg \tau$, where λ is the magnetic length and $D \sim \rho^{-1} \sim \lambda^2\tau^{-1}$ (for definiteness we mean the case of short-range disorder in the lowest Landau level). Clearly, within the scaling approach the average conductance is scale independent. Indeed, whatever the function $\mathrm{P}(x)$ is, the velocity-velocity correlator $G_v(t) = \int d^2\mathbf{r} r^2 \frac{\partial^2 P(r,t)}{\partial t^2}$ is proportional, within the scaling description, to $\delta(t)$. Hence, it is corrections to the scaling that determine the asymptotic behavior of $G_v(t)$ at $t \gg \tau$. We expect that the quantum kinetic correlations decay at the transition in a power-law manner:

$$G_v(t) \sim \frac{D}{\tau}\left(\frac{\tau}{t}\right)^{1+\frac{y}{2}} . \qquad (12)$$

We now appeal to Eq. (9) in order to find y. The essential idea is that the tail in $G_v(t)$ originates from a power-law (in $\tau/t \ll 1$) correction to the limiting value of $K(t)$ at $t \to \infty$. Indeed, at the critical point $K(t)$ at $\Delta \to 0$ is parameterized as a function of the single variable t/τ. It follows that the r-dependent correlators written in the scaling form as functions of r^2/Dt satisfy Eq. (9) only if $K(t)$ is strictly constant. Hence, taking into account t-dependent terms in $K(t)$ will generate corrections to the scaling. As noted above, these yield the sought tail in $G_v(t)$. We thus expand $K(t)$ in τ/t and substitute the leading term in the expansion into Eq. (9). Multiplying the equation by r^2 and doing the integrals over r, we obtain the contribution to $G_v(t)$ which is due to the dispersion of $K(t)$ – it is given by Eq. (12) with

$$y = \eta . \qquad (13)$$

The exponent y defined by Eq. (12) governs behavior of the average conductance $g(L)$ in a finite sample of size $L \times L$. Specifically, the conductance at $L \to \infty$ is given by the Kubo formula for the conductivity $g(\infty) = \frac{\pi}{2}\rho\int_0^\infty dt G_v(t)$, and cutting the integral over t at $t \sim L^2/D$ yields $g(L)$. Since we associate the long-term tail in $G_v(t)$ with the decay of the difference $K(\infty) - K(t)$, which in turn is given by the behavior of $P(0,t)$ [Eq. (10)], we finally arrive at Eq. (2).

A cautionary remark is in order at this point. We assumed that the corrections to $P(r,t)$ and $M(r,t)$ do not cancel each other in Eq. (9) exactly unless other irrelevant scaling fields are taken into account – since, apart from the common exponent y, the corrections depend on microscopic details (say the correlation properties of disorder). Eq. (9) thus tells us that there *is* a contribution to $G_v(t)$ which scales with $y = \eta$. However, because of

the parametric correlations in the r.h.s., we cannot rule out solely on the basis of this equation the possibility that there exists another contribution with smaller y (a "less irrelevant" scaling field), not at all related to the behavior of $K(t)$. Therefore, to be precise, what we have been able to prove is the *inequality* $y \leq \eta$. At this point we might seek support from numerical data. These suggest strongly that the sign $\leq$ above is in fact "equals": indeed, the values of $y \simeq 0.4 - 0.5$ [11,12] and $\eta \simeq 0.4$ [4,11] obtained in the numerical simulations are fairly close to each other.

So far we have dealt with the scaling of the *average* conductance $g(L)$. Of what significance could it be in real experiments in view of the fact that at the critical point the mesoscopic fluctuations are very strong? To extract the scale dependent δg at the QH transition, the dephasing length $L_\phi \sim (D/T)^{1/2}$ (or, interchangeably, $L_\omega \sim (D/\omega)^{1/2}$ in ac measurements) should be much smaller than the sample size – the *measured* conductance then will be a self-averaging well-defined quantity $g^{\rm eff}(L_\phi)$ (independent of L in $2d$) with a small temperature dependent correction $\delta g^{\rm eff}(L_\phi) \sim (\lambda/L_\phi)^y$. In this respect, the experiment with the QH metal looks much like the measurement of the localization corrections in a good metal. Note two points relevant to the experiment. First, the average ("coherent") conductance $g(\infty)$ (at $L/\lambda \to \infty$) and the measured at finite T ("effective") conductance $g^{\rm eff}(\infty)$ (at both $L_\phi/\lambda \to \infty$ and $L/L_\phi \to \infty$) are by no means the same. One way to look at the problem is to divide the system at the critical point into blocks of size L_ϕ and couple them to each other "incoherently", thus representing the system as a classical resistance network which obeys the Ohm's law locally. Since the quantum interference is preserved inside the blocks, the elementary conductances g_i of this network will fluctuate strongly around mean $g(L_\phi)$. One may now consider expanding $g^{\rm eff} - g$ in series in the cumulants of g_i [13] – the first term in the expansion is given by $-(\langle g_i^2\rangle - \langle g_i\rangle^2)/2\,\langle g_i\rangle$. Since there is no small parameter in this expansion, it follows immediately that $|g^{\rm eff}(\infty) - g(\infty)| \sim 1$. What is important to us, however, is that the scaling of the corrections to either of the quantities is governed by the same exponent y. The difference between the measured and averaged conductances is nonetheless crucial when referring to the concept of the universal g at the quantum phase transition (in this respect the above arguments complement those in [14]). Second, the applicability of the non-interacting model considered in the paper to the experiment at the integer QH critical point requires comment. There exists a long-range contribution to the velocity-velocity correlator $G_v(t)$ associated with electron-electron interactions: as is well known, at large g in $2d$ it decays as t^{-1} [3] – i.e., it scales similarly to the weak-localization contribution. We have shown that the t^{-1} tail which is due to the localization effects is transformed at the integer QH transition into $t^{-1-\eta/2}$. The question as to what extent the Coulomb interaction affects this behavior of $G_v(t)$ requires further study.

In conclusion, we have studied how the critical conductance $g(L)$ at the metal-insulator transition behaves as a function of the system size L. We relate the size dependent correction $\delta g(L) \sim (l/L)^y$ to the quantum return probability and argue that the exponent y is the same as η – the critical exponent of eigenfunction correlations.

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